Key

Math 4 | Name______________________
U3 L3 I1 Activity | Date________

In this activity, you are working towards the following learning goals:

- I can interpret expressions for rules of rational functions that model problem conditions

Coffee and Cream Anyone?

A typical coffee mug will hold around 300 cubic centimeters (cc) of coffee. This will leave space for cream. The coffee situation is this: what percent of a coffee and cream mixture is coffee? This is sometimes called the strength of the coffee. Many restaurants provide cream in small containers. Since the containers are usually not full, an estimation of the amount of cream in one container is 6 cc. Suppose you put one container of cream in your coffee, the strength has changed from 100% to something lower. The strength can be found by dividing the amount of coffee by the amount of mixture; thus the mixture now has a strength of $\frac{300}{300+6}=0.980$, or 98% coffee. If you add two containers of cream, the strength has changed to $\frac{300}{300+6\cdot 2}=0.962$, or 96.2% coffee.

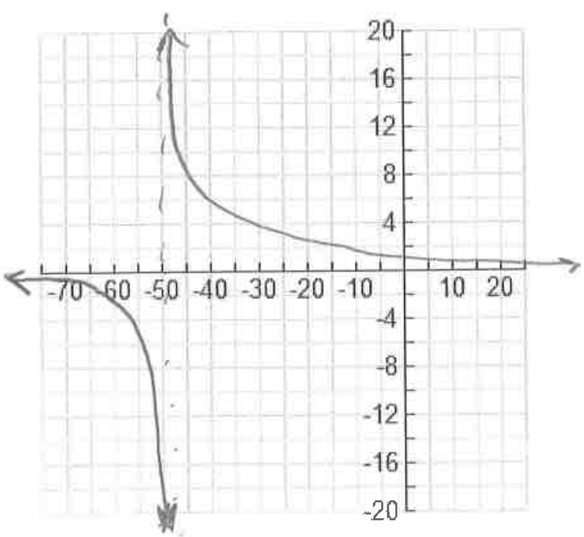
1. Write a formula that will give you the strength of the coffee for x number of creams.

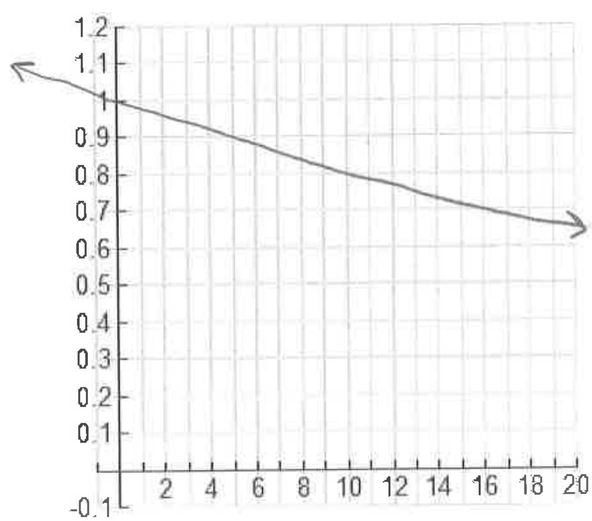
$$S_c = \frac{300}{300+6x}$$

2. If you have not done so in question (1), simplify your formula by factoring out a common term.

$$S_c = \frac{300}{6(50+x)} = \frac{50}{50+x}$$

3. Graph your function from question (2) on your calculator. Sketch it using both windows shown below.





4. Which graph is the more practical graph, given the context of the problem? Explain.

The second one... no negative values!

5. What happens to the function S_c when $x = -50$. Explain both in terms of the graph and the equation.

There is a vertical asymptote. If $x = -50$, the denominator will $= 0$.

The function S_c is a **rational function**. A rational function is the quotient of polynomials. The parent function for all rational functions is $f(x) = \frac{1}{x}$. In your previous math courses, you have learned that division by zero is undefined. This means in the function $\frac{1}{x}$, the value of x cannot be zero. If the domain of the function contains 0, the value of the function is undefined at that value. The definition of a function requires that the elements of the domain must be real numbers AND the corresponding values of the range must also be real numbers; thus $x \neq 0$.

In the rational function $\frac{1}{x}$, the behavior of the function as values of x get closer and closer to 0 proves to be one of the interesting characteristics of the function. To investigate the behavior of the parent rational function $\frac{1}{x}$, a numeric representation is developed below.

x	-500	-10	-5	-1	-0.1	-0.01	-0.001	0.001	0.01	0.1	1	2	5	10	500
$\frac{1}{x}$	-0	-0.1	-0.2	-1	-10	-100	-1000	1000	100	10	1	0.5	0.2	0.1	0.002

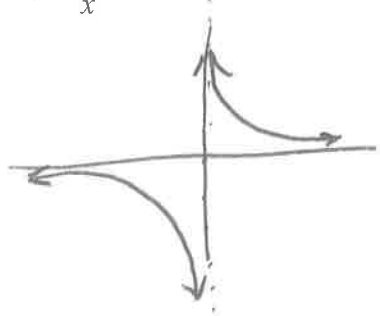
6. What is happening to the values in the above table as x approaches 0 *from the left*? That is:

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

7. What is happening to the values in the above table as x approaches 0 *from the right*? That is:

$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

8. Sketch a graph of $f(x) = \frac{1}{x}$. Explain how your answers to questions (6) and (7) show up in the graph of $\frac{1}{x}$.



Moving right, up to $X = 0$, the graph shoots downward without bound. Moving left toward zero, the graph increases without bound.

9. Consider the function $C(x) = \dfrac{113x}{100 - x}$

a. State the **domain** of C.

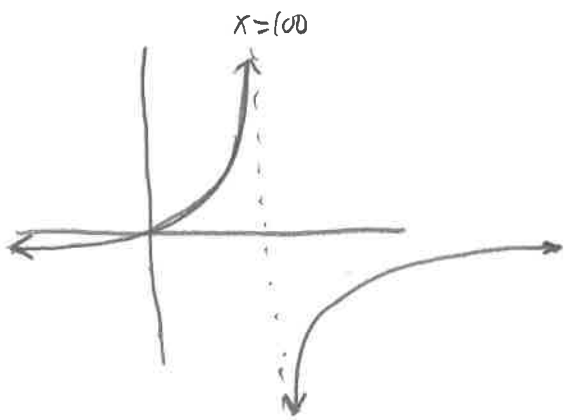
$\{x : x \neq 100\}$

b. Graph the function on your screen using the window:

$$[-200, 300] \; by \; [-600, 600]$$

$$x_{scl} = 30 \qquad y_{scl} = 100$$

Sketch your graph below.



c. $\lim_{x \to 100^-} C(x) = \infty$

d. $\lim_{x \to 100^+} C(x) = -\infty$

e. Write the equation of the vertical asymptote.

$x = 100$

Now let's study this function's **application**.

$C(x)$, the cost (in thousands of dollars) of removing x % of a city's pollutants discharged into a lake.

$$C(x) = \frac{113x}{100 - x}$$

h. Now state the domain that fits the above practical application.

$0 \leq X < 100$ % are always between 0 & 100 inclusive, except here, $x \neq 100$.

i. Find $C(50)$ and **interpret in the context of the problem.**

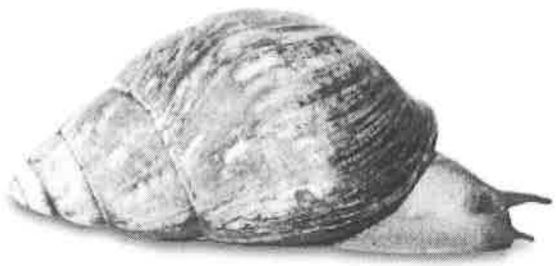
$C(50) = \frac{113(50)}{100-50} = 113$ It would cost \$130K to remove 50% of the city's pollutants.

j. Interpret the result from part (c) in the context of the problem.

It would get really expensive to remove close to 100% of the city's pollutants.

10. In the modern world, nearly everyone carries (and sometimes drops) a variety of small and fragile electronic devices like cell phones, calculators, personal digital assistants, and music players. Engineers who design those tools work hard to come up with devices that can withstand repeated drops before breaking. On the other hand, there are engineering problems in which the goal is to drop an object so that it (or a target on the ground) *will* break.

One of the most intriguing natural examples of dropping with intent to break is exhibited by sea gulls and crows who feed on mollusks that have shells, like snails and clams. Biologists have observed a species of crows that pick up *whelks*, lift them into the air, and drop them on rocks to break open the shells.



What has especially intrigued biologists who observe the whelk-dropping behavior of northwestern crows is the uncanny way that they seem to rise consistently to a height of about 5 meters before dropping the shells onto the ground.

When the crow picks up and drops the Whelk, the shell does not always break on the first drop. Therefore, the crow must balance the amount of effort it takes to fly into the air with the Whelk with the likelihood that the Whelk shell will break when dropped from a certain height. The average work required (in joules) for the crow to break the Whelk shell at height h is:

$$W(h) = \frac{30h^2 + 573h}{10h - 9}$$

a. What is the domain of W?

$10h - 9 = 0$

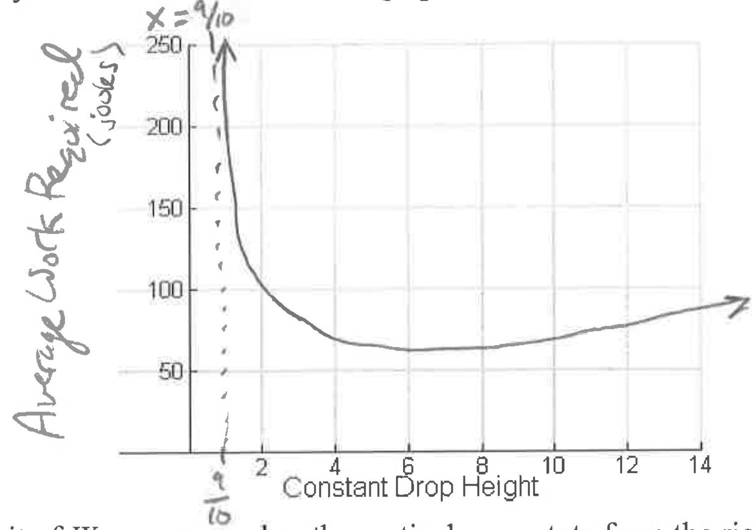
$10h = 9$

$h = \frac{9}{10}$ $\rightarrow$ $\boxed{\{h : h \neq \frac{9}{10}\}}$

b. Write the equation of the vertical asymptote of W.

$X = \frac{9}{10}$ $\updownarrow$

c. Graph W on your calculator and sketch a graph on the below axis. Label the vertical asymptote.



d. Find the limit of W as x approaches the vertical asymptote from the right. Write your answer in limit notation.

$$\lim_{x \to \frac{9}{10}^+} W(h) = \infty$$

e. Interpret the limit and vertical asymptote in the context of the problem.

As the crow's height above ground gets closer to $\frac{9}{10}$ m, the amount of work it has to do to break the shell increases without bound.

X Use your calculator to find the optimal drop height for the crow. How does that compare with the actual drop height the crows use?

Find minimum value: (4.478, 88.526)

Close to 5 m!

~~SUMMARY:~~ Look back at the learning goals for this activity. Write a few sentences describing what you learned about rational functions in terms of the learning goals.